00 P2 A5	**5A** •1 $3(2\cos^2 x° - 1)$ •2 $6\cos^2 x° + \cos x° - 2 = 0$ •3 $(2\cos x° - 1)(3\cos x° + 2)$ •4 •5 •4 $\cos x° = \frac{1}{2}$, $x = 60,\ 300$ •5 $\cos x° = -\frac{2}{3}$, $x = 132,\ 228$ **ans:** 60, 131·8, 228·2, 300
01 P1 Q5	**5C** •1 $2\sin x°\cos x°$ •2 $\cos x°(2\sin x° - 1)$ •3 $\cos x° = 0,\ \sin x° = \frac{1}{2}$ •4 $90, 30, 150$ •3 $\sin x° = \frac{1}{2}$ and $x = 30, 150$ •4 $\cos x° = 0$ and $x = 90$ •5 $\left(150, -\frac{\sqrt{3}}{2}\right)$
01 P1 Q7	**2C, 5C** •1 $\sin\left(x + \frac{\pi}{4}\right)$ •2 $\cos\left(x + \frac{\pi}{4}\right)$ •3 $\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}$ *and complete* •4 $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$ •5 $\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) - \left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)$ •6 $\frac{2}{\sqrt{2}}\sin x$ •7 $x = \frac{\pi}{4}, \frac{3\pi}{4}$
02 P1 Q3	**2C, 5C**

	•1 $\sin(2x°)$ •2 $2\sin(x°)$ •3 $2\sin(2x°)=2\sin(x°)$ •4 appearance of $2\sin(x°)\cos(x°)$ •5 $2\sin(x°)(2\cos(x°)-1)$ •6 $\sin(x°)=0$ *and* 0, 180, 360 •7 $\cos(x°)=\frac{1}{2}$ *and* 60, 300 *or* •6 $\sin(x°)=0$ *and* $\cos(x°)=\frac{1}{2}$ •7 0, 60, 180, 300, 360
4.(JAN) 02 P2	**4C** •1 *use eg* $\cos 2x=1-2\sin^2 x$ •2 $\sin^2 x=\frac{1}{4}$ •3 $\sin x=\frac{1}{2}\ldots\ldots x=\frac{\pi}{6},\frac{5\pi}{6}$ •4 $\sin x=-\frac{1}{2}\ldots\ldots x=\frac{7\pi}{6},\frac{11\pi}{6}$ •3 $\sin x=\frac{1}{2}\ldots\ldots x=\frac{\pi}{6}$ **and** $\sin x=-\frac{1}{2}\ldots\ldots x=\frac{7\pi}{6}$ •4 $\frac{5\pi}{6},\frac{11\pi}{6}$ ans: $\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ **OR** •1 *use eg* $2\sin^2 x=1-\cos 2x$ •2 $\cos 2x=\frac{1}{2}$ •3 $2x=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3}$ •4 $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$ **Alternative for •3 and •4** •3 $2x=\frac{\pi}{3},\frac{5\pi}{3}$ **and** $x=\frac{\pi}{6},\frac{5\pi}{6}$ •4 $2x=\frac{7\pi}{3},\frac{11\pi}{3}$ **and** $x=\frac{7\pi}{6},\frac{11\pi}{6}$ **OR** •1 *use eg* $\cos 2x=\cos^2 x-\sin^2 x$ •2 $\tan^2 x=\frac{1}{3}$ •3 $\tan x=\frac{1}{\sqrt{3}}\ldots\ldots x=\frac{\pi}{6},\frac{7\pi}{6}$ •4 $\tan x=-\frac{1}{\sqrt{3}}\ldots\ldots x=\frac{5\pi}{6},\frac{11\pi}{6}$
03 P2 Q10	**5A** •1 $3(2\cos^2(x)-1)\ldots\ldots$ •2 $6\cos^2(x)+10\cos(x)-4=0$ •3 $2(3\cos(x)-1)(\cos(x)+2)$ •4 $\cos(x)=\frac{1}{3}$ *and* $\cos(x)=-2$ •5 $x=1.23$ *and* no solution
05 P2 Q8	**5A**

	•1 $k\sin(2x)=\sin(x)$ •2 $k\times 2\sin(x)\cos(x)$ •3 $\sin(x)\left(2k\cos(x)-1\right)=0$ •4 $\sin(x)=0$ ••• $\cos(x)=\frac{1}{2k}$ •5 $\sin(x)=0 \Rightarrow x=0,\pi,2\pi$ i.e. at (O),B and D ••• $\cos(x)=\frac{1}{2k}$ is for A and C. •
06 P1 Q7	**4C** •1 $\sin(x^\circ)-2\sin(x^\circ)\cos(x^\circ)=0$ •2 $\sin(x^\circ)\left(1-2\cos(x^\circ)\right)=0$ •3 $\sin(x^\circ)=0$ *or* $\cos(x^\circ)=0.5$ •4 $x=0,180,360,\quad 60,300$ **OR** •3 $\sin(x^\circ)=0$ *and* $x=0,180,360$ •4 $\cos(x^\circ)=0.5$ *and* $x=60,300$
07 P1 Q6	**4C** •1 $2\sin(x^\circ)\cos(x^\circ)$ •2 $\cos(x^\circ)\left(2\sin(x^\circ)-6\right)=0$ •3 $\cos(x^\circ)=0$ *and* $x=90,270$ •4 $\sin(x^\circ)=3$ *and no solution* *or* •3 $\cos(x^\circ)=0$ *and* $\sin(x^\circ)=3$ •4 $x=90,270$ *and no solution* **OR** •1 $2\sin(x^\circ)\cos(x^\circ)$ •2 *either* $\cos(x^\circ)=0$ *or* $\cos(x^\circ)\neq 0$ *stated explicitly* •3 $\cos(x^\circ)=0\Rightarrow x=90$ *or* 270 •4 $2\sin(x^\circ)=6\Rightarrow$ *no solution*
08 P2 Q5	**5B** •1 $\cos 2x=1-2\sin^2 x$ •2 $3\sin^2 x-2\sin x-1=0$ •3 $(3\sin x+1)(\sin x-1)=0$ •4 $\sin x=-\frac{1}{3}$ | •5 $\sin x=1$ •5 $199.5^\circ, 340.5^\circ$ | 90°
10 P2 Q4	know to use double angle formula express as quadratic in $\cos x$ start to solve	•1 $2\times(2\cos^2 x-1)\ldots$ •2 $4\cos^2 x-5\cos x-6$ •3 $(4\cos x+3)(\cos x-2)$ } $=0$ must appear at these lines to gain

	reduce to equations in $\cos x$ only complete solutions to include only one where $\cos x = k$ with $\lvert k \rvert > 1$	$\bullet^4$ $\cos x = -\frac{3}{4}$ and $\cos x = 2$ $\bullet^5$ $2{\cdot}419,\ 3{\cdot}864$ and no solution **or** $\bullet^4$ $\cos x = 2$ and no solution $\bullet^5$ $\cos x = -\frac{3}{4}$ and $2{\cdot}419,\ 3{\cdot}864$
11 P1 Q23	$\bullet^1$ $2\cos^2 x^\circ - 1\ldots$ **stated, or implied by** $\bullet^2$ $\bullet^2$ $2\cos^2 x^\circ - 3\cos x^\circ + 1$ $\bullet^3$ $(2\cos x^\circ - 1)(\cos x^\circ - 1)$ $= 0$ must appear at either of these lines to gain $\bullet^2$. $\bullet^4$ $\cos x^\circ = \frac{1}{2}$ and $\cos x^\circ = 1$ $\bullet^5$ 0, 60 and 300 Candidates who include 360 lose $\bullet^5$ **or** $\bullet^4$ $\cos x^\circ = 1$ and $x = 0$ $\bullet^5$ $\cos x^\circ = \frac{1}{2}$ and $x = 60$ or 300 Candidates who include 360 lose $\bullet^4$ $\bullet^6$ $2x = 0$ and 60 and 300 $\bullet^7$ 0, 30, 150, 180, 210 and 330	
12 P2 Q6	condition on u_n coefficient connect coefficient with given interval	$\bullet^1$ $-1 < \sin x < 1$ $\bullet^2$ in interval, $0 < \sin x < 1$
	appropriate limit method substitute for limit	$\bullet^3$ limit $= \frac{\cos 2x}{1 - \sin x}$ $\bullet^4$ $\frac{1}{2}\sin x = \frac{\cos 2x}{1 - \sin x}$
	use appropriate double angle formula express in standard form start to solve quadratic equation reduce to equations in $\sin x$ only select valid solution	$\bullet^5$ $\ldots 1 - 2\sin^2 x \ldots$ $\bullet^6$ e.g. $3\sin^2 x + \sin x - 2$ $\bullet^7$ e.g. $(3\sin x - 2)(\sin x + 1)$ $= 0$ must appear to gain $\bullet^6$. $\bullet^8$ $\sin x = \frac{2}{3}$ or $\sin x = -1$ $\bullet^9$ $x = 0{\cdot}730$ or outwith interval

13 P2 Q8		**Method 1**
	use correct double angle formulae	$\bullet^1$ $2\sin x\cos x$
	form correct equation	$\bullet^2$ $2\sin x\cos x - 2\cos^2 x = 0$
	take out common factor	$\bullet^3$ $2\cos x(\sin x - \cos x) = 0$
	proceed to solve	$\bullet^4$ $\cos x = 0$ and $\sin x = \cos x$
	find solutions	$\bullet^5$ $\frac{\pi}{2}$ $\quad$ $\bullet^6$ $\frac{3\pi}{2}$
	find remaining solutions	$\bullet^6$ $\frac{\pi}{4}$ $\quad$ $\frac{5\pi}{4}$
		Method 2
	use double angle formula	$\bullet^1$ $\cos 2x + 1$
	form correct equation	$\bullet^2$ $\sin 2x - \cos 2x = 1$
	express as a single trig function	$\bullet^3$ $\sqrt{2}\sin\left(2x - \frac{\pi}{4}\right) = 1$
	proceed to solve	$\bullet^4$ $\sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$
	find solutions	$\bullet^5$ $2x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$ $\quad$ $\bullet^6$ $\frac{9\pi}{4}, \frac{11\pi}{4}$
	find solutions	$\bullet^6$ $x = \frac{\pi}{4}, \frac{\pi}{2}$ $\quad$ $x = \frac{5\pi}{4}, \frac{3\pi}{2}$
14 P2 Q6	use correct double angle formula	$\bullet^1$ $\sin x - 2\left(1 - 2\sin^2 x\right)$ stated or implied by $\bullet^2$
	arrange in standard quadratic form	$\bullet^2$ $4\sin^2 x + \sin x - 3 = 0$
	start to solve	$\bullet^3$ $(4\sin x - 3)(\sin x + 1) = 0$
	reduce to equations in $\sin x$ only	$\bullet^4$ $\sin x = \frac{3}{4}$ and $\sin x = -1$
	process to find solutions in given domain	$\bullet^5$ $0{\cdot}848$, $2{\cdot}29$ and $\frac{3\pi}{2}$
		OR
		$\bullet^4$ $\sin x = \frac{3}{4}$ and $x = 0{\cdot}848,\ 2{\cdot}29$
		$\bullet^5$ $\sin x = -1$, and $x = \frac{3\pi}{2}$

PRE 2000 ANSWERS – **Trig Equations (Degrees)**

1	$\bullet^1$ substitute $1-2\sin^2 x°$ for $\cos 2x°$ $\bullet^2$ substitute $1-\sin^2 x°$ for $\cos^2 x°$ $\bullet^3$ $3\sin^2 x° + 2\sin x° - 1 = 0$ $\bullet^4$ $(3\sin x° - 1)(\sin x° + 1) = 0$ $\bullet^5$ $\sin x° = \frac{1}{3},\ -1$ $\bullet^6$ $19{\cdot}5,\ 160{\cdot}5,\ 270$
2	$\bullet^1$ $2\sin x\cos x + \sin x = 0$ $\bullet^2$ $\sin x(2\cos x + 1) = 0$ $\bullet^3$ $\sin x = 0,\ \cos x = -\frac{1}{2}$ $\bullet^4$ 1*st*: $x = 0,\ 180$ $\bullet^5$ 2*nd*: $x = 120,\ 240$
3	$\bullet^1$ substitute $2\cos^2 x° - 1$ for $\cos 2x°$ $\bullet^2$ $(2\cos x° - 1)(\cos x° + 1) = 0$ $\bullet^3$ $\cos x° = \frac{1}{2},\ \cos x° = -1$ $\bullet^4$ $x = 60, 300$ $\bullet^5$ $x = 180$
4	$\bullet^1$ Replacing $\cos 2x$ by $2\cos^2 x - 1$ $\bullet^2$ $2\cos^2 x + 5\cos x - 3 = 0$ $\bullet^3$ $(2\cos x - 1)(\cos x + 3) = 0$ $\bullet^4$ $60°$ $\bullet^5$ $300°$ **and** no extraneous solutions **and** no solution for $\cos x = -3$ indicated. [If a reason is given, it must be valid].
5	$\bullet^1$ $\cos 2x° = 1 + \sin x°$ $\bullet^2$ $2\sin^2 x° + \sin x° = 0$ $\bullet^3$ $\sin x° = 0$ *or* $-\frac{1}{2}$ $\bullet^4$ $x = 210$
6	(*a*) $\bullet^1$ strategy: ie $\sin 2x = 2\sin x\cos x$ $\bullet^2$ $\sin x = 0$ AND $\cos x = \frac{1}{3}$ $\bullet^3$ 0, 180 AND 360 $\bullet^4$ 70.5 AND 289.5 AND no other angles (*b*) $\bullet^5$ $f(x) = 2\sin x°,\ g(x) = 3\sin 2x°$ (*c*) $\bullet^6$ $x = 70.5$ AND 289.5 $\bullet^7$ $y = 1.89$ AND -1.89 (*d*) $\bullet^8$ 70.5 AND 180 $\bullet^9$ 289.5 AND 360 $\bullet^{10}$ use inequality signs logically to connect the points of intersection (ie **not** for $180 < x < 70.5$)

PRE 2000 ANSWERS - **Trigonometric Equations (Radians)**

1

- $\bullet^1$ subst. leading from $\sin^2$ *to* $\cos^2$
- $\bullet^2$ $-6\cos^2 a - \cos a + 6 = 5$
- $\bullet^3$ solving the quadratic
- $\bullet^4$ 1.231 **and** 2.094

2

Method 1

use correct double angle formulae	$\bullet^1$	$2\sin x \cos x$
form correct equation	$\bullet^2$	$2\sin x \cos x - 2\cos^2 x = 0$
take out common factor	$\bullet^3$	$2\cos x(\sin x - \cos x) = 0$
proceed to solve	$\bullet^4$	$\cos x = 0$ and $\sin x = \cos x$
find solutions	$\bullet^5$	$\frac{\pi}{2}$ | $\frac{3\pi}{2}$
find remaining solutions	$\bullet^6$	$\frac{\pi}{4}$ | $\frac{5\pi}{4}$

Method 2

use double angle formula	$\bullet^1$	$\cos 2x + 1$
form correct equation	$\bullet^2$	$\sin 2x - \cos 2x = 1$
express as a single trig function	$\bullet^3$	$\sqrt{2}\sin\left(2x - \frac{\pi}{4}\right) = 1$
proceed to solve	$\bullet^4$	$\sin\left(2x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$
find solutions	$\bullet^5$	$2x - \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$ | $\frac{9\pi}{4}, \frac{11\pi}{4}$
find solutions	$\bullet^6$	$x = \frac{\pi}{4}, \frac{\pi}{2}$ | $x = \frac{5\pi}{4}, \frac{3\pi}{2}$

3

(*a*)
- $\bullet^1$ $(180 - 2x)°$
- $\bullet^2$ $\frac{\sin x°}{p} = \frac{\sin(180 - 2x)°}{r}$
- $\bullet^3$ $\sin(180 - 2x)° = \sin 2x°$ **stated explicitly**

(*b*)
- $\bullet^4$ $60°$
- $\bullet^5$ $\sin x° = \sin 2x°$
- $\bullet^6$ $\sin x°(2\cos x° - 1) = 0$
- $\bullet^7$ $\sin x° = 0$ *and* $\cos x° = \frac{1}{2}$
- $\bullet^8$ $x = 60$ is only answer **stated explicitly**

4

condition on u_n coefficient	$\bullet^1$	$-1 < \sin x < 1$
connect coefficient with given interval	$\bullet^2$	in interval, $0 < \sin x < 1$

	appropriate limit method	$\bullet^3$ limit $= \dfrac{\cos 2x}{1-\sin x}$
	substitute for limit	$\bullet^4$ $\frac{1}{2}\sin x = \dfrac{\cos 2x}{1-\sin x}$
	use appropriate double angle formula	$\bullet^5$ $\ldots 1-2\sin^2 x \ldots$
	express in standard form	$\bullet^6$ e.g. $3\sin^2 x+\sin x-2$ } $=0$ must appear
	start to solve quadratic equation	$\bullet^7$ e.g. $(3\sin x-2)(\sin x+1)$ } to gain $\bullet^6$.
	reduce to equations in $\sin x$ only	$\bullet^8$ $\sin x = \frac{2}{3}$ or $\sin x = -1$
	select valid solution	$\bullet^9$ $x = 0{\cdot}730$ or outwith interval
5	(*a*) $\bullet^1$ correct scales $\bullet^2$ zeros $\bullet^3$ graph of $y = \sin \frac{\pi t}{6}$ $\bullet^4$ graph of $y = -\sin \frac{\pi t}{6}$ (*b*) $\bullet^5$ indication of translation $\begin{pmatrix} 0 \\ 1.1 \end{pmatrix}$ to $y = -\sin \frac{\pi t}{6}$ $\bullet^6$ for minima at $W = 0.1$` $\bullet^7$ sketch (*c*) $\bullet^8$ indicate on graph effect of fire at $t = 23$ $\bullet^9$ $t = 26$ (± 1) $\bullet^{10}$ about July($\pm$1) 1990	
6	$\bullet^1$ $2\cos^2\theta - 1 + 8\cos\theta + 9$ $\bullet^2$ $2(\cos\theta + 2)^2 = 0$ or "$b^2 - 4ac$" $= 16 - 4\times 1\times 4$ $\bullet^3$ $\cos\theta = -2$ twice or "$b^2 - 4ac$" $= 0$	$\bullet^4$ $\cos\theta = -2$ has no solution